

University of Dundee

## Dundee Discussion Papers in Economics 106

Molana, Hassan

*Publication date:*  
1999

[Link to publication in Discovery Research Portal](#)

*Citation for published version (APA):*

Molana, H. (1999). *Dundee Discussion Papers in Economics 106: Effectiveness of fiscal policy in a model of imperfect competition with transactions money*. (Dundee Discussion Papers in Economics; No. 106). University of Dundee.

### General rights

Copyright and moral rights for the publications made accessible in Discovery Research Portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from Discovery Research Portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain.
- You may freely distribute the URL identifying the publication in the public portal.

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.



University  
of  
Dundee

---

---

# Dundee Discussion Papers in Economics

---

---

## Effectiveness of Fiscal Policy in a Model of Imperfect Competition with Transactions Money

Hassan Molana

Department of  
Economic Studies,  
University of Dundee,  
Dundee.  
DD1 4HN

Working Paper  
No. 106  
January 1999  
ISSN:1473- 236X

# **Effectiveness of Fiscal Policy in a Model of Imperfect Competition with Transactions Money**

Hassan Molana

January 1999

**ABSTRACT:** This paper examines the effectiveness of fiscal policy in a general equilibrium macromodel with transactions money and an oligopolistic product market. The results suggest that although money may be neutral and play no direct role as policy instrument, its indirect impact on the effectiveness of fiscal policy can be quite substantial. In particular, *(i)* fiscal policy becomes ineffective as the weight attached to money is reduced; *(ii)* the fiscal multiplier becomes negative when the elasticity of substitution between money and leisure exceeds unity; and *(iii)* it is possible that policy effects are in fact enhanced as the product market becomes more competitive.

**KEYWORDS:** transactions cost; elasticity of substitution; oligopolistic competition; fiscal multiplier; crowding out

**JEL CLASSIFICATION:** H3

**Correspondence:** Department of Economics, University of Dundee, Dundee DD1 4HN, UK.  
Tel: (00)44-(0)1382-344375; Fax: (00)44-(0)-1382-344691;  
e-mail: h.h.molana@dundee.ac.uk

## 1. Introduction

There are now quite a few studies that construct relatively simple macromodels with precise microfoundations in order to examine the possible links that may exist between market distortions and effectiveness of macroeconomic policies. These models depart, in one way or another, from the '*Walrasian*' tradition and are designed to exhibit '*New Keynesian*' features. For instance, Dixon (1987), Mankiw (1988) and Startz (1989) all illustrate that characterising the product market by a standard oligopolistic or monopolistic structure is sufficient to yield a larger tax financed fiscal multiplier<sup>1</sup>. More recently, a number of studies have looked at the robustness of this result. For example, Molana and Moutos (1992) show that the fiscal multiplier can be zero or even negative depending on which component of personal income is taxed; Dixon and Lawler (1996) illustrate that the above result is rather sensitive to the specification of households' preferences; and Torregrosa (1998) provides an example in which there is a negative relationship between the fiscal multiplier and firm's market power.

The purpose of this paper is to extend the above analysis by investigating the effectiveness of fiscal policy in an economy with transactions money. The main role of money in the conventional, perfect competition, textbook macromodels is to provide liquidity services. In addition, when agents have perfect foresight, prices adjust rapidly and markets clear instantaneously, money plays a neutral role. It therefore does not provide a suitable policy instrument for affecting output and employment. The indirect role of money, nevertheless, is not quite clear and it is desirable to examine how the existence of (neutral) transactions money can influence the effectiveness of fiscal policy indirectly. We therefore analyse how the tax financed fiscal multiplier is affected by the agents' propensity to hold money for transactions purposes. The analysis is carried out within a framework which postulates that consumers associate transactions cost with purchasing consumption goods. The transactions technology is described by a *subjective* cost function which depends positively on consumption and negatively on the real balances and the potential leisure time available. The latter are thought to facilitate transactions and are assumed to appear in the cost function with a constant elasticity of substitution with respect to each other. Our results suggest that while money maintains its neutrality, its existence influences the policy effectiveness in three ways. First, it is found that fiscal policy loses its effectiveness as the weight attached to money is reduced. Second, it is shown that the fiscal multiplier becomes negative if the elasticity of substitution between money balances and leisure exceeds unity. Finally, the relationship between the magnitude of the fiscal multiplier and firms' market

power is found to be non-monotonic and it is possible that policy effects are in fact enhanced as the product market becomes more competitive.

The rest of the paper proceeds as follows. Section 2 outlines the production sector and describes firms' behaviour within a standard oligopolistic framework. Section 3 explains households' behaviour and obtains the demand functions for goods and real balances and the labour supply function. Section 4 derives the tax financed fiscal multiplier and explains how the transmission process works towards crowding out the initial impact of an exogenous expansion of aggregate demand. Section 5 concludes the paper.

## 2. Firms

Given the purpose of the paper, the product market is kept at its simplest form and a homogeneous good is assumed to be produced by a finite number of identical firms<sup>2</sup>. The representative firm, denoted by subscript  $j=1,...,n$ , chooses its output  $y_j$  to maximise profits

$$\Pi_j = Py_j - W\ell_j, \quad (1)$$

where  $P$  is the price level determined on the market demand function  $P = P(y)$ ,  $y = \sum_{j=1}^n y_j$  is total output,  $W$  is the nominal wage rate, and  $\ell$  is labour which is assumed to be used as the only input. Assuming constant returns to scale and normalising the marginal product of labour to unity, the production function is  $y_j = \ell_j$  which can be substituted in equation (1) to give  $\Pi_j = (P-W)y_j$ . When firms are identical, follow a Cournot-Nash strategy, and face a demand function with unit price elasticity, maximum profit is achieved by a mark-up or price setting rule  $P = kW$ , where  $k > 1$  and is a decreasing function of number of firms,  $n$ . In particular, the product market structure tends to perfect competition or monopoly as  $k \rightarrow 1$  or  $k \rightarrow \infty$ <sup>3</sup>. Finally, aggregating (1) over firms and letting  $\Pi = \sum_{j=1}^n \Pi_j$  and  $\ell = \sum_{j=1}^n \ell_j$ , we obtain

$$Py = W\ell + \Pi, \quad (2)$$

which shows that aggregate income consists of wages and profits.

### 3. Households

In common with other studies, the population of households is kept constant and normalised to unity. The representative household is assumed to choose values of consumption  $c$ , real money balances  $m$ , and leisure time  $h$  to maximise its objective function – described below – subject to the following budget constraint

$$Pc + M = (1-t)[W(1-h) + \Pi] + M_o, \quad (3)$$

where  $M$  is the desired money stock and  $m = M/P$ ,  $t$  is the income tax rate,  $1-h$  is the time spent at work, or labour supply (the time endowment is normalised to unity), profits  $\Pi$  are accrued to households as dividend income, and  $M_o$  is the initial endowment of money.

The above set up implies that the households' objective function should contain  $c$ ,  $h$ , and  $m$  as its arguments, and the purpose of the paper will then be to examine the consequences, for the effectiveness of fiscal policy, of including real money balances amongst households' choice variables. One way to have such an objective function is to postulate a utility function which depends on consumption, leisure, as well as the real money holdings. However, there is a well known debate that casts doubt on this approach since such a utility function is considered as a reduced form equation approximating households' preferences<sup>4</sup>. As a result, this approach will not allow us to identify the explicit role of the real money balances in the analysis. To avoid this problem, we shall first define separately functions that represent (i) the disutility – or subjective cost – of the transactions activity, and (ii) the utility of consumption, and then augment these explicitly to construct an objective function for households.

To represent the disutility of transactions, we postulate a simple transactions technology by defining a subjective cost function,  $Z(c, m, h)$ , which is twice continuously differentiable in all its arguments. In order to constitute a plausible cost function,  $Z$  ought to be increasing in  $c$  but decreasing in  $m$  and  $h$  since the latter are thought to facilitate transactions; money acts as a catalyst and the availability of time reduces the tediousness of the underlying activity. Hence, imposing the normalisation  $Z(0, m, h) = 0$  for all  $m > 0$  and  $h > 0$ , one would expect  $Z(c, m, h) > 0$  for all  $c > 0$ ,  $m > 0$  and  $h > 0$ , and the partial derivatives of  $Z$  to satisfy  $Z_c > 0$ ,  $Z_{cc} < 0$ ,  $Z_m < 0$ ,  $Z_{mm} > 0$ ,  $Z_h < 0$ ,  $Z_{hh} > 0$ . As for the cross partial derivatives, while consistency requires  $Z_{cm} < 0$  and  $Z_{ch} < 0$ ,  $Z_{mh}$  should be allowed to assume positive, zero as well as negative values so as to accommodate a sufficiently flexible substitutability between  $m$  and  $h$ . The following function satisfies these requirements

$$Z(c, m, h) = \left( \frac{c}{\left( \gamma m^{1-1/s} + h^{1-1/s} \right)^{\frac{1}{1-1/s}}} \right)^{1-\alpha}, \quad (4)$$

where  $\alpha$ ,  $s$  and  $\gamma$  are positive constant parameters,  $0 < \alpha < 1$ ,  $s (\neq 1)$  is the elasticity of substitution between  $m$  and  $h$ , and  $\gamma > 0$  is a scale parameter which reflects the weight attached to money relative to the time factor,  $h$ . If we define the utility of consumption by  $V(c)=c$ , we can augment  $V$  and  $Z$  and construct an objective function for households' as follows

$$U(c, m, h) = \log(V) + \log(Z) = \alpha \log(c) + \left( \frac{1-\alpha}{1-1/s} \right) \log(\gamma m^{1-1/s} + h^{1-1/s}), \quad (5)$$

When we impose the restriction  $s=1$  on the above, the corresponding preferences may be approximated by the following Cobb-Douglas utility function

$$U(c, m, h) = \alpha \log(c) + \left( \frac{(1-\alpha)\gamma}{1+\gamma} \right) \log(m) + \left( \frac{1-\alpha}{1+\gamma} \right) \log(h), \quad (5)'$$

which imposes a unit elasticity of substitution between each pair of  $c$ ,  $m$  and  $h$ . Note also that in both (5) and (5)' the utility function tends to a Cobb-Douglas in  $c$  and  $h$  as  $\gamma$  approaches zero, and hence the results become comparable with those obtained by studies which do not explicitly include money, e.g. Mankiw (1988) and Startz (1989).

Writing the budget constraint in (3) as

$$c + m + \omega h = (N + M_o)/P, \quad (3)'$$

where  $\omega = (1-t)W/P$  is the after tax, or disposable, real wage and  $N$  denotes the *potential* disposable nominal income,

$$N = (1-t)(W + \Pi), \quad (6)$$

the demand functions associated with maximising (5) subject to (3)' are

$$c = \alpha \left( \frac{N + M_o}{P} \right), \quad (7)$$

$$h = \left( \frac{(1-\alpha)(\gamma\omega)^{-s}}{1+\gamma^{-s}\omega^{1-s}} \right) \left( \frac{N+M_o}{P} \right), \quad (8)$$

$$m = \left( \frac{1-\alpha}{1+\gamma^{-s}\omega^{1-s}} \right) \left( \frac{N+M_o}{P} \right). \quad (9)$$

#### 4. General equilibrium and policy effectiveness

Given that money is only held by households for transaction purposes, when the stock of money in the economy is kept constant monetary equilibrium requires  $m = M_o/P$ . Imposing this condition, equation (9) implies that the money stock determines the nominal value of households' *potential* disposable income,  $N$ ,

$$N = \left( \frac{\alpha + \gamma^{-s}\omega^{1-s}}{1-\alpha} \right) M_o. \quad (10)$$

Substituting (10) in (7) and (8) gives the demand functions for  $c$  and  $h$  when the economy is in monetary equilibrium,

$$c = \left( \frac{\alpha(1+\gamma^{-s}\omega^{1-s})}{1-\alpha} \right) \left( \frac{M_o}{P} \right), \quad (11)$$

$$h = (\gamma\omega)^{-s} \left( \frac{M_o}{P} \right). \quad (12)$$

Thus, it is clear from the above equations that consumption, labour supply and the real value of potential disposable income are neutral with respect to the money stock and are only affected by the government's fiscal policy instrument,  $t$  and firms' market power,  $k$  which are implicit in  $\omega$  if and only if  $s \neq 1$ .

Starting from a monetary equilibrium position, the economy sketched above can be described by the following equations,

$$y = \beta \left( 1 + \gamma^{-s} k^{s-1} (1-t)^{1-s} \right) \left( \frac{M_o}{P} \right) + \frac{G}{P}, \quad (13)$$



$$\ell = 1 - \gamma^{-s} k^{s-1} (1-t)^{-s} \left( \frac{M_o}{W} \right), \quad (14)$$

$$P = kW, \quad (15)$$

$$y = \ell, \quad (16)$$

and

$$\frac{G}{P} = ty. \quad (17)$$

Equation (13) is the aggregate demand function equating output with the sum of private and public demand for goods, that is  $y = c + g$ , where  $g = G/P$  is the government demand for goods<sup>5</sup> and  $c$  is substituted out using (11). Note that we have also set  $\beta = \frac{\alpha}{1-\alpha}$  and have made use of  $\omega \equiv (1-t)(W/P) = (1-t)/k$ . Equation (14) is the labour supply equation given by  $\ell = 1 - h$  where  $h$  is substituted out using (12) and introducing nominal wage explicitly by making use of the mark-up rule,  $W = P/k$ . The latter appears in equation (15) to express the firms' price setting rule. Equation (16) is the aggregate production function and equation (17) gives the government budget constraint which equates public expenditure  $g = G/P$  with the tax revenue<sup>6</sup>  $t[W(1-h) + \Pi]/P$ .

Equations (13)-(17) can be solved to yield the equilibrium values of  $P$ ,  $W$ ,  $t$ ,  $y$  and  $\ell$  for any given exogenous levels of  $G$  and  $M_o$ , the parameters  $\beta$ ,  $\gamma$  and  $s$ , and the degree of competition in the product market measured by  $k$ . First, however, we note that money preserves its neutrality since a proportional change in  $G$ ,  $M_o$ ,  $P$  and  $W$  leaves employment and output, and its components  $c$  and  $g$ , unaffected. This result holds regardless of the size of the elasticity of substitution,  $s$ . In contrast,  $s$  turns out to play a crucial role in transmitting the effect of a tax financed fiscal expansion. The corresponding multiplier

$$\frac{dy}{dg} = \frac{1}{\bar{t} + \theta} = \left( \frac{1}{\bar{t}} \right) \left( 1 - \frac{\theta}{\bar{t} + \theta} \right), \quad (18)$$

where  $\bar{t}$  denotes the initial equilibrium value of  $t$  and<sup>7</sup>

$$\theta = \left( \frac{\beta(1-\bar{t})}{1-s} \right) \left( 2k^{-1} + \gamma^s(1-\bar{t})^{s-1}k^{-s} + \gamma^{-s}(1-\bar{t})^{1-s}k^{s-2} \right), \quad (19)$$

and it can be easily verified that:  $t+\theta > 0$  when  $s < 1$ ;  $t+\theta < 0$  when  $s > 1$ ;  $|t+\theta| > 1$  for all  $s > 0$ ;  $|t+\theta| \rightarrow \infty$  as  $s \rightarrow 1$  or as  $\gamma \rightarrow 0$ ;  $t+\theta \rightarrow -\infty$  as  $s \rightarrow \infty$ ; and that monotonicity of  $dy/dg$  with respect to  $k$  is likely to break down as  $s$  exceeds 2 sufficiently.

The multiplier in (18) is the long-run<sup>8</sup> or the balanced budget multiplier but it is also expressed such that the impact effect and the crowding out effect of the policy are made explicit. The impact effect is<sup>9</sup>  $(1/\bar{t})$  and is multiplied by the *crowding out adjustment factor* which reduces the size of the multiplier to below unity and can also make it negative if  $\theta < 0$ . Given that the sign and size of the multiplier determine the effectiveness of fiscal policy, the main results concerning the indirect role of transactions money may be summarised as follows:

- (i) The sign of the multiplier is determined by the size of the elasticity of substitution between money and hours of leisure;  $dy/dg$  is positive, zero or negative as  $s < 1$ ,  $s = 1$  or  $s > 1$ .
- (ii) When  $s \neq 1$ ,  $|dy/dg| < 1$  and the impact of policy becomes smaller as the weight attached to money in the utility function reduces;  $|dy/dg| \rightarrow 0$  as  $\gamma \rightarrow 0$ .
- (iii) The impact of a fiscal expansion is reduced as money and leisure become perfect substitutes;  $dy/dg \rightarrow 0$  as  $s \rightarrow \infty$ .
- (iv) The impact of policy is enhanced as the product market becomes more competitive when  $s$  is sufficiently large.

Figure 1 below shows<sup>10</sup> how the multiplier is affected by  $s$  as well as by  $k$ ,  $\gamma$  and  $\beta$ . The curve labelled **A** constitutes the benchmark with which other cases are compared. Curve **B** shows the effect of a rise in  $k$  by doubling the number of firms and reveals the non-monotonicity of the multiplier with respect to firms' market power since the size of the multiplier shrinks as  $k$  rises when  $s$  becomes sufficiently large (where **B** intersects **A** from below). Curve **C** illustrates the impact of a fall in  $\gamma$ , the weight attached to money in the CES bundle comprising money and leisure. Finally, curve **D** depicts the impact of a fall in  $\beta$ , the weight given to consumption relative to the CES bundle in the augmented objective function.

**Figure 1 Here**

It is possible to illustrate the above results by considering the transmission mechanism following an exogenous stimulation of aggregate demand, in particular by examining the way prices and tax rate adjust to bring the economy back to equilibrium. As an example, we show below the relevance of the elasticity of substitution in a standard diagram describing the economy as it is summarised by equations (13)-(17). First consider the case  $s=1$  which is depicted in Figure 2 below.

### Figure 2 Here

The product market is shown in  $(y, P)$  space where the curve labelled **AD** is the aggregate demand function in equation (13). The labour market is shown in  $(\ell, W)$  space where the curve labelled **LS** is the labour supply function in (14). Firms' price setting or mark-up rule in (15) is given in  $(W, P)$  space and the production function in (16) is shown in  $(\ell, y)$  space. The solid and broken lines show the two situations before and after the expansion, respectively. The initial equilibrium is traced by solid lines which show the levels of  $P$ ,  $W$ ,  $y$  and  $\ell$  for a given  $t$  and  $g$ . The effects of a tax-financed expansion are illustrated by broken lines. The rise in government expenditure shifts **AD** to the right. This exogenous stimulation of aggregate demand raises demand for labour, pushing up nominal wages. Firms, experiencing a reduction in their profit margin, use the mark-up rule to adjust the price level and retain their profit margin intact. On the one hand, the rise in price crowds out private consumption and reduces demand. On the other hand, the rise in the tax rate required to finance the expansion reduces the disposable wage and hence labour supply also falls as shown by the leftward shift in the **LS** curve. Note from equation (13) that when  $s=1$  the change in the tax rate does not have any further effect on the position of **AD**. Thus, what happens in this case is a purely proportional adjustment of the price level, the wage rate and the tax retention rate -  $P$ ,  $W$  and  $(1-t)$  - which is sufficient to guarantee  $dy=0$ , and  $dg=ydt$ . The only effect of this policy therefore is 100% crowding out of private consumption, since  $dc=-dg$ .

Now consider the cases  $s<1$  and  $s>1$  which are shown, respectively, in Figures 3 and 4 below. In these figures, as in Figure 2, the solid lines indicate the initial situation and the broken lines refer to the final position. The dotted lines show the intermediate situation. First consider Figure 3 which depicts the case  $s<1$  and hence a positive multiplier. The rise in government spending is shown by shifting the **AD** curve in  $(y, P)$  space to the right. The new position is depicted by the dotted curve and the corresponding rise in demand for goods is

shown to lead to a rise in demand for labour and hence a higher nominal wage rate  $W$ , which firms absorb by marking up their price. As the government finances this expansion by taxation both the  $AD$  and the  $LS$  curves shift to their new positions shown by the broken curves. The new equilibrium is traced by the broken lines and shows that although there is some partial crowding out the level of output is nevertheless higher than its original level. This result follows because when  $s < 1$  the shift in the  $LS$  curve is relatively small and hence  $W$  does not need to rise too much. As a result, the rise in  $P$  is also relatively small and the private component of aggregate demand does not fall excessively. In contrast, when  $s > 1$  the shift in the  $LS$  curve is sufficiently large, as depicted in Figure 4, to give rise to a relatively high price level and induce an excessive reduction in private demand for goods, sufficient to give rise to more than 100% crowding out.

### **Figures 3 & 4 Here**

#### **5. Summary and conclusions**

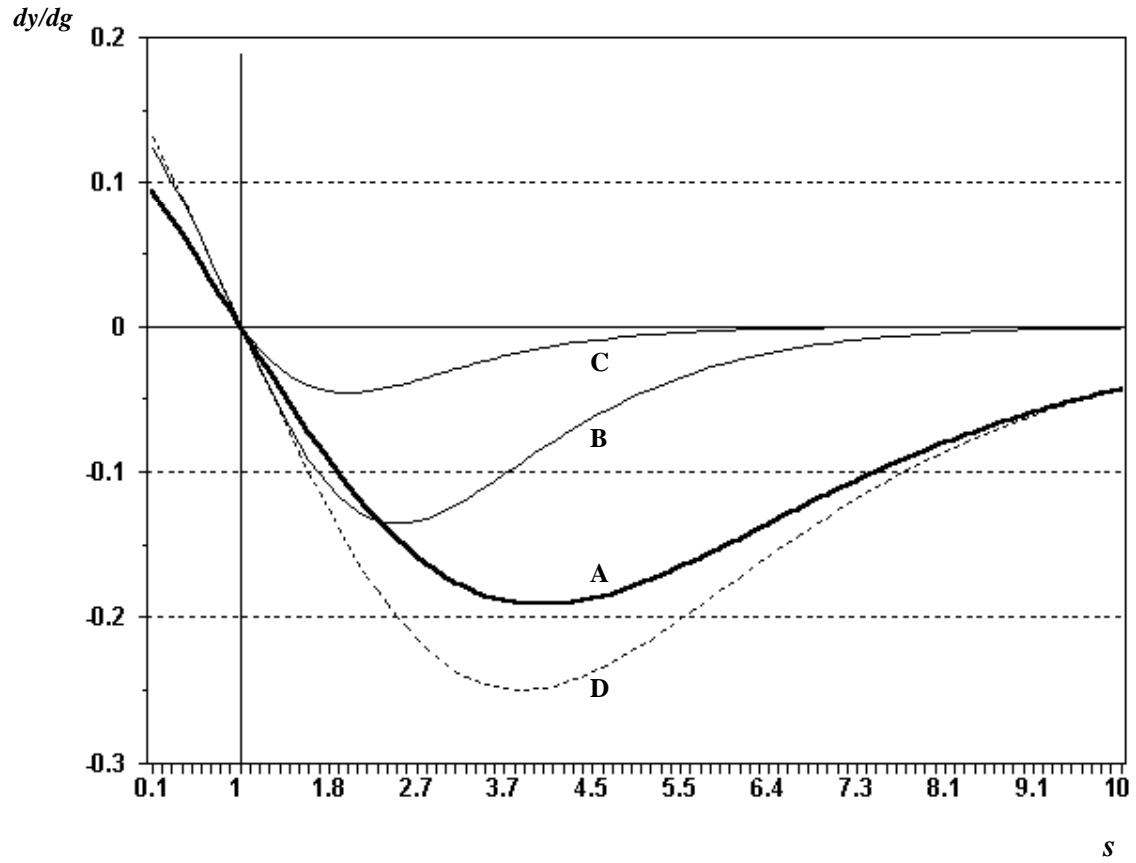
This paper has examined whether the existence of transactions money in a macromodel with imperfectly competitive goods market can be instrumental in transmitting the effects of a fiscal expansion. The analysis is carried out within a framework in which the availability of money and time is essential for transactions – since money acts as a catalyst and the availability of time reduces the tediousness of the underlying activity – and real money balances and leisure are regarded as substitutes in facilitating the transactions activity. When labour market is competitive, changes in the real disposable wage rate determine how leisure, consumption and real money balances are substituted to ensure optimality. As a result, the weight given to money relative to leisure as well as the elasticity of substitution between these are found to play important roles in transmitting the fiscal policy effects.

## References

- Azariadis, C. and Smith, B.D. (1996), "Private information, money, and growth: indeterminacy, fluctuations, and the Mundell-Tobin effect", *Journal of Economic Growth*, 1, 309-332.
- Blanchard, O. and Kiyotaki, N. (1987), "Monopolistic competition and the effects of aggregate demand", *American Economic Review*, 77, 647-666, reprinted in Mankiw and Romer (1991).
- Dixon, H. (1987), "A simple model of imperfect competition with Walrasian features", *Oxford Economic Papers*, 39, 134-160.
- Dixon, H. and Lawler, P. (1996), "Imperfect competition and the fiscal multiplier", *Scandinavian Journal of Economics*, 98, 219-231.
- Dixon, H. and Rankin, N. (1995), *The new macroeconomics; imperfect competition and policy effectiveness*, Cambridge University Press, Cambridge.
- Feenstra, R. (1986), "Functional equivalence between liquidity costs and the utility of money", *Journal of Monetary Economics*, 17, 271-291.
- Lawler, P. (1997), "The real balance effect and the non-neutrality of money in a simple model of a small open economy", *Bulletin of Economic Research*, 49, 1-15.
- Mankiw, G. (1988), "Imperfect competition and the Keynesian Cross", *Economics Letters*, 26, 7-13, reprinted in Mankiw and Romer (1991).
- Mankiw, G. (1992), "The reincarnation of Keynesian economics", *European Economic Review*, 36, 559-65.
- Mankiw, G. and Romer, D. (1991), *New Keynesian Economics; imperfect competition and sticky prices*, The MIT Press, Cambridge, Massachusetts.

- Molana, H. and Moutos, T. (1989), “Useful government expenditure in a simple model of imperfect competition” in Reider, U., Gessner, P., Peyerimoff, A. and Radermacher, F.J. (eds.), *Methods of Operations Research*, 63, Verlag Anton Hain, Frankfurt.
- Molana, H. and Moutos, T. (1992), “A note on taxation, imperfect competition and the balanced budget multiplier”, *Oxford Economic Papers*, 43, 68-74.
- Nishimura, K. (1992) *Imperfect Competition, Differential Information, and Microfoundations of Macroeconomics*, Clarendon Press, Oxford University Press.
- Silvestre, J. (1993), “The market-power foundations of macroeconomic policy”, *Journal of Economic Literature*, 31, 105-41.
- Startz, R. (1989), “Monopolistic competition as a foundation for Keynesian macroeconomic models”, *Quarterly Journal of Economics*, 104, 737-52.

Figure 1. Role of  $s$ ,  $k$ ,  $\gamma$  and  $\beta$  in determining the sign and size of the multiplier



A:  $\beta=3$  ( $\alpha=3/4$ ),  $\gamma=1$ ,  $k=4/3$  ( $n=4$ );      B:  $\beta=3$  ( $\alpha=3/4$ ),  $\gamma=1$ ,  $k=2$  ( $n=2$ );  
C:  $\beta=3$  ( $\alpha=3/4$ ),  $\gamma=1/2$ ,  $k=4/3$  ( $n=4$ );      D:  $\beta=2$  ( $\alpha=2/3$ ),  $\gamma=1$ ,  $k=3/4$  ( $n=4$ );

Figure 2. The transmission process following a tax financed fiscal expansion;  $s=1$

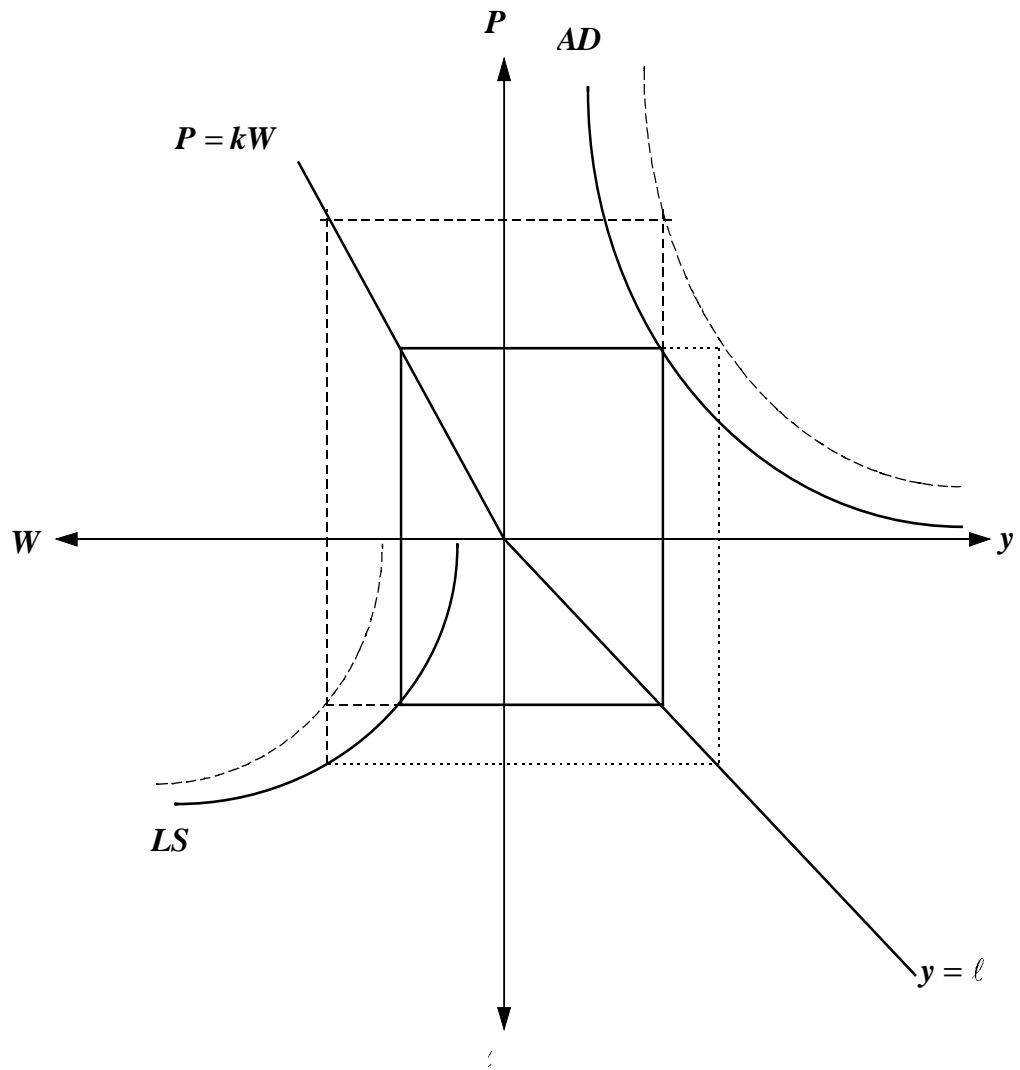




Figure 3. The transmission process following a tax financed fiscal expansion;  $s < 1$

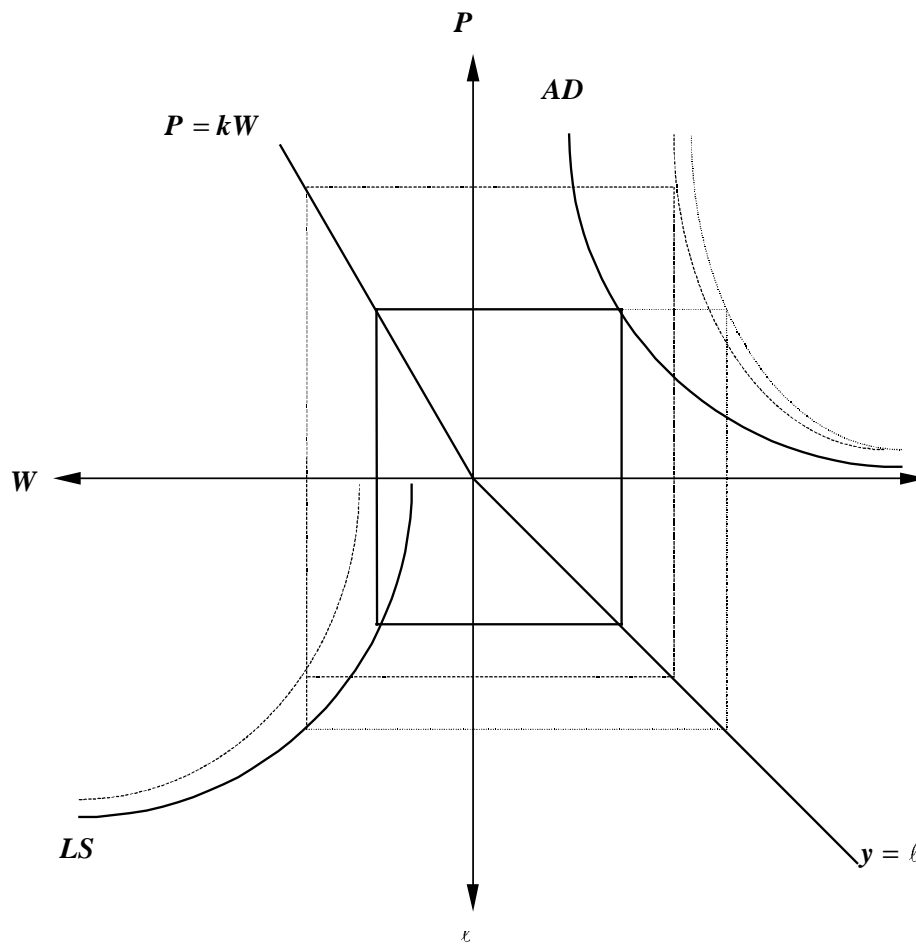
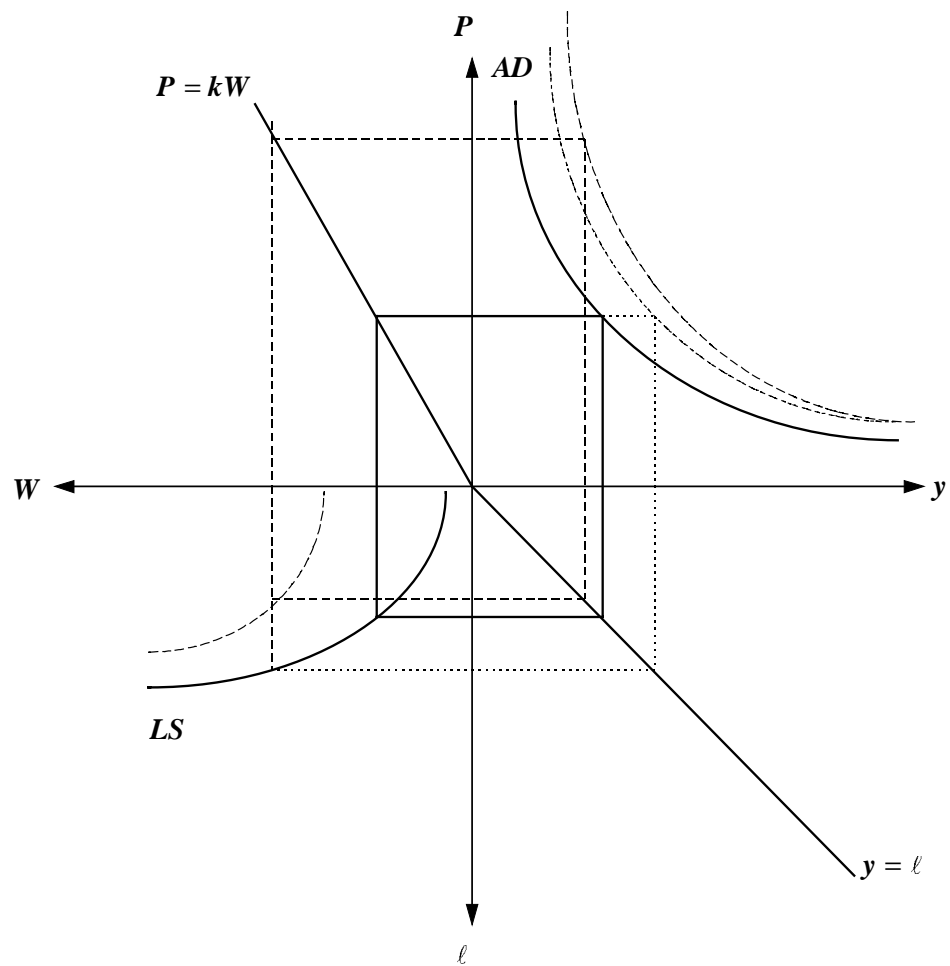


Figure 4. The transmission process following a tax financed fiscal expansion;  $s > I$



### Appendix: Derivation of the multiplier

This appendix derives the multiplier expression given by equation (18) in the paper. The multiplier is related to the solution of equations (13)-(17) which can be reduced to the following three equations

$$y = g + \beta \left( 1 + \gamma^{-s} ((1-t)/k) \right)^{1-s} \left( \frac{M_o}{P} \right), \quad (\text{A1})$$

$$y = 1 - (\gamma(1-t)/k)^s \left( \frac{M_o}{P} \right), \quad (\text{A2})$$

$$g = ty. \quad (\text{A3})$$

(A1) and (A2) give the equilibrium conditions in the product and labour markets and (A3) is the government budget constraint. Solving equations (A1) and (A2) for  $y$  we obtain

$$y = \frac{g + \beta \left( \frac{1-t}{k} + \left( \frac{\gamma(1-t)}{k} \right)^s \right)}{1 + \beta \left( \frac{1-t}{k} + \left( \frac{\gamma(1-t)}{k} \right)^s \right)}. \quad (\text{A4})$$

(A4) gives the equilibrium level of  $y$  for any  $t$  and  $g$ , and (A3) provides that combination of  $y$ ,  $t$  and  $g$  which satisfy the government budget constraint. For algebraic tractability we define

$$f = \beta \left( \frac{1-t}{k} + \left( \frac{\gamma(1-t)}{k} \right)^s \right), \quad (\text{A5})$$

and rewrite (A4) as

$$y = \frac{g + f}{1 + f}. \quad (\text{A6})$$

Totally differentiating (A3) and (A6) with respect to  $y$ ,  $t$  and  $g$ , we have

$$tdy + ydt = dg,$$

and

$$dy = \frac{(1+f)(dg + f'dt) - (g+f)f'dt}{(1+f)^2} = \frac{(1+f)dg + (1-g)f'dt}{(1+f)^2},$$

where

$$f' = \frac{df}{dt} = -\beta \left( \frac{1}{k} + \frac{s}{1-t} \left( \frac{\gamma(1-t)}{k} \right)^s \right). \quad (\text{A7})$$

Solving these for  $dy$  we obtain

$$dy = \frac{(1+f)dg + (1-g)f'(1/y)(dg - tdy)}{(1+f)^2},$$

and hence the multiplier,

$$\frac{dy}{dg} = \frac{1+f + (1-g)(1/y)f'}{(1+f)^2 + (1-g)(t/y)f'} = \frac{1}{t + \frac{1+f-t}{1 + \left( \frac{1-g}{1+f} \right) \left( \frac{1}{y} \right) f'}}.$$

or simply

$$\frac{dy}{dg} = \frac{1}{t + \theta}, \quad (\text{A8})$$

where

$$\theta = \frac{1+f-t}{1 + \left( \frac{1-g}{1+f} \right) \left( \frac{1}{y} \right) f'}, \quad (\text{A9})$$

Finally, substituting for  $f$  and  $f'$  from (A5) and (A7), (A11) can be written as

$$\theta = \left( \frac{\beta(1-t)}{1-s} \right) \left( 2k^{-1} + \gamma^s (1-t)^{s-1} k^{-s} + \gamma^{-s} (1-t)^{1-s} k^{s-2} \right), \quad (\text{A10})$$

It can be easily verified that:  $t+\theta > 0$  when  $s < 1$ ;  $t+\theta < 0$  when  $s > 1$ ;  $|t+\theta| > 1$  for all  $s > 0$ ;  $|t+\theta| \rightarrow \infty$  as  $s \rightarrow 1$  or as  $\gamma \rightarrow 0$ ;  $t+\theta \rightarrow -\infty$  as  $s \rightarrow \infty$ ; and that monotonicity of  $dy/dg$  with respect to  $k$  is likely to break down as  $s$  exceeds 2 sufficiently. These conform to the graphs in Figure 1 and support statements (i)-(iv).

## Notes

<sup>1</sup> See Dixon and Lawler (1996) and Molana and Moutos (1992) on the generality of the results obtained by these studies and Mankiw (1992) for a general discussion. Further examples can be found in Mankiw and Romer (1991), Nishimura (1992), and Dixon and Rankin (1995) among others. Silvestre (1993) and Dixon and Rankin (1995) provide detailed surveys.

<sup>2</sup> Generalising this market structure by introducing product differentiation does not change the main results. See Blanchard and Kiyotaki (1987) and Startz (1989) for examples.

<sup>3</sup> The first order condition for profit maximisation can be written as

$$\frac{\partial \Pi_j}{\partial y_j} = P + y_j (dP / dy) (\partial y / \partial y_j) - W = P [1 - (y_j / y) (\partial y / \partial y_j) (1 / \varepsilon)] - W = 0$$

where  $\varepsilon = -(P / y) (dy / dP)$  is the price elasticity of demand. The above result is obtained by setting  $\varepsilon = 1$  and  $(\partial y / \partial y_j) = 1$ , and noting that the ratio  $y_j / y$  is the inverse of the number of firms  $n$ . Hence,  $W/P = 1/k$  as required, where  $k = 1/(1-1/n)$ . The assumption of unit elastic demand is clarified below.

<sup>4</sup> See Feenstra (1986) for details.

<sup>5</sup> Here we treat government spending as in traditional macromodels and do not consider any externalities. See Molana and Moutos (1989) and Startz (1989) for a discussion of ‘*useful*’ government expenditure. The analysis will not be affected if we allow for substitution between private and public expenditure by replacing  $V(c) = c$  with  $V(c, g) = (g^\delta) c$  where  $\delta > 0$  ( $< 0$ ) if  $g$  is Edgworth complement (Edgworth substitutes) with respect to  $c$ .

<sup>6</sup> Note, from equation (2), that  $(W(1-h) + \Pi) / P = (W\ell + \Pi) / P = y$ .

<sup>7</sup> Calculations underlying (18) and (19) are rather tedious and hence eliminated but they are available from the author on request.

<sup>8</sup> The concept of long-run here refers to the total multiplier after the tax rate is adjusted to pay for the initial public deficit caused by a fiscal expansion and does not have any implication for the market structure which is assumed to remain intact by some unspecified exogenous barrier to entry as, for example, in Mankiw (1988). See Dixon and Lawler (1996) for an example of allowing the long-run to correspond to a change in market structure to ensure zero profits.

<sup>9</sup> Note that from the government budget constraint in (17) that before the tax rate is changed  $\frac{dy}{dg} = \frac{1}{t}$ .

<sup>10</sup> Graphs in Figure 1 are obtained by simulating the model defined by equations (13)-(17) and the values of the multiplier plotted are calculated as  $\Delta y / \Delta g$ .